Macroeconomic instability and microeconomic financial fragility: a stock-flow consistent approach with heterogeneous agents

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Abstract

This paper introduces heterogeneous microeconomic behavior into a demand-driven stock-flow consistent model, in order to study the joint dynamics of leverage, income distribution and aggregate demand. The distinctive feature is in that the aggregation of heterogeneous agents is not performed numerically as in traditional agent-based models but by means of an innovative analytical methodology, originally developed in statistical mechanics and recently imported into macroeconomics. The numerical analysis of the dynamical system reveals that a faster expansion of the financial sector, relatively to the real sector, can sustain growth even with a rising inequality and stagnating demand from wage earners. The side effects of the financialization are higher leverage and volatility of fluctuations. Financialization and leverage are also significantly impacted by the degree of heterogeneity in firms’ investment decisions.

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1 Introduction

The 2008 financial turmoil itself, and the process of de-leveraging by the private sector observed in the following years, have drawn the attention to the crucial role of credit as a factor leading both to the instability of the system and to a strengthening of real-financial linkages in the economy. This view, which was central to the work of Hyman Minsky, is also supported by the vast historical evidence presented in Schularick and Taylor (2012), which highlights that credit booms tend to be followed by deeper recessions when compared to other financial crises episodes. The macroeconomic framework proposed by this paper introduces micro-foundations into the stock-flow consistent modeling approach in order to shed light on the relationships between private sector leverage, income distribution, aggregate demand and the instability of the economic system. The theoretical model involves heterogeneous firms aggregated by means of an innovative analytical methodology.

The stock-flow consistent (SFC) models, first developed by Tobin (1969) and Godley and Lavoie (2007), have recently received renewed attention (Khalil and Kinsella, 2011; Bezemer, 2010, among others). By taking into account all flows of income between different sectors in the economy as well as their accumulation into financial and tangible assets, this type of models are able to trace the flows of credit and stock of debt that could help predict the crisis (Godley, 1999). Besides allowing for formal Minskyan analyses of corporate debt and financial fragility (Dos Santos, 2005), SFC models have recently been used to study the macroeconomic effects of shareholder value orientation and financialization (Treeck, 2009), as well as that of household debt accumulation (Kim and Isaac, 2010).

The limitation of the SFC approach is that of modeling economic behavior in aggregative terms, thus excluding the heterogeneity of agents as a source of financial instability. The relevance of a microeconomic analysis in modeling financial fragility is stressed by Minsky: “an ultimate reality in a capitalist economy is the set of interrelated balance sheets among the various units” (Minsky, 2008, 116). Taylor and O’Connel (1985) remark
that “shifts of firms among classes as the economy evolves in historical time underlie much of its cyclical behavior. This detail is rich and illuminating but beyond the reach of mere algebra”. This lack of proper modeling tools is one of the reasons why the vast majority of the literature of Minskyan inspiration, including SFC models, is formulated in aggregative terms. The recent development of new computational and analytical tools has now made feasible the solution of macroeconomic models with heterogeneous agents.

Agent-based modeling is based on the idea that any aggregate economic system is more than the sum of the microeconomic decisions of (rational) agents and aims to study the emerging properties of decentralized microeconomic interactions taking place in complex and adaptive economic and financial systems. As argued by Delli Gatti et al. (2010), agent-based models can outperform traditional ones in explaining a wide variety of aggregate phenomena such as fluctuating growth, bankruptcy chains and firms’ sizes and growth rates distributions. In particular, such models can be found to be very useful for the analysis of financial instability, which clearly requires an understanding of the economy as an “out-of-equilibrium” system incorporating heterogeneous economic behavior (Delli Gatti et al., 2005, 2010).

The agent-based and the SFC approaches to economic modeling can be thought of as complementary in their understanding of the crucial role of real-financial linkages for the instability of the economic system, as well as its macroeconomic dynamics. Nevertheless, the fact that agent-based models can only be solved numerically has two main drawbacks. The first one is that, while the micro-behavioral rules are defined and modeled, there is no analytical definition for the relationships between macro and micro-variables. Second, as a consequence the causality links within the system cannot be clearly identified.

In this context, this paper introduces heterogeneous microeconomic behavior into a demand-driven SFC model. The model developed here is composed of firms, households and a financial sector. Firms have heterogeneous degrees of financial soundness.

The distinctive feature is in that the aggregation of heterogeneous agents is performed by means of an innovative analytical methodology originally developed in statistical mechanics and recently imported into macroeconomics (see Aoki and Yoshikawa, 2006; Di Guilmi, 2008; Foley, 1994; Weidlich, 2000, among others). This modeling approach builds from the idea that, as the economy is populated by a very large number of dissimilar agents, an analytical model cannot keep track of the conditions of every single agent at
each point in time. As Aoki and Yoshikawa (2006) remark: “the point is that precise behavior of each agent is irrelevant. Rather we need to recognize that microeconomic behavior is fundamentally stochastic.” Therefore, a microfounded analytical model should look at how many agents are in a certain condition, rather than at which agents, and represent their evolution in probabilistic terms. This approach is particularly suitable to microfound SFC models, since it is able to endogenously derive the macro-equations and the dynamics of flows from the microeconomic behavioral rules, without imposing ad-hoc constraints.

In order to implement this method in the stock-flow consistent approach, we first simulate the model as agent based with full heterogeneity of firms. These simulations serve the only purpose of generating the values of the variables that will be subsequently used for the simulation of the system of flows, as done in standard stock-flow consistent models.

The proposed framework has two main objectives. First, the dynamical system will present the analytical links between the financial micro-variables and the macroeconomy, in order to study the joint dynamics of leverage, market capitalization, income distribution and aggregate demand.

Second, the theoretical structure can assess the effects of the interaction between leverage dynamics and income inequality, by studying the shift of households between classes of income along the business and leverage cycle.

Whereas in standard macro-models the evolution of the economy is determined by aggregate common shocks, in our framework the idiosyncratic micro-shocks drive the phase transitions of the system. The analysis thus can set the stage for micro and macro policy experiments that will be developed in a further stage.

2 The model

The economy described in this paper is composed by firms, households and a financial sector. As in the conventional neo-Kaleckian literature, prices are set as a mark-up over labor costs, investment behavior is determined independently and the degree of capacity utilization of firms adjusts to the quantity they sell. The mark-up and the functional distribution of income are assumed to (exogenously) depend on the degree of industrial concentration and the relative bargaining power of workers and capitalists.

Firms are divided into two classes and switch between them. While hedge
firms finance all their investment with internal resources, \textit{borrowing} firms finance part or all their investment with stocks and/or bonds. The method proposed in Section 3 will allow the share of firms in each class to affect macroeconomic dynamics. There is no microfoundation for the household sector, which is treated as an aggregate with two types of income: wage and profits.

Households allocate their wealth between money and firms’ shares. Interest rates on bonds are assumed to be set exogenously by the Central Bank, with the price of firms’ shares being determined by supply and demand in the market for stocks, rather than flows, of these shares. Capital gains (or losses) then affect consumption levels via wealth effects, thus allowing for the study of the role of asset price booms and bursts in aggregate demand. Finally, the financial sector is considered as an aggregate: its basic role is to provide loans, hence holding debt (or bonds) as an asset, and to create money deposits endogenously as liabilities.

\section{2.1 The Firms}

A single firm is identified by the superscript \( j \), while its state or group by the subscript \( z = 1, 2 \). Thus when a variable is written as \( x_1^j \), it refers to the firm \( j \) belonging to state 1; a variable with only the subscript indicates the mean-field value (the average value for the units in the group). Symbols without superscript or subscript refer to aggregate variables. The numbers of firms in each group are indicated by \( N_1 \) and \( N_2 \) with \( N_1(t) + N_2(t) = N(t) \).

Firms prefer to finance their investment with internal resources they have previously accumulated in the form of money \( M^j \) and the flow of retained profits \( A^j \). If these are not sufficient they issue stocks and bonds. Accordingly, we can define two classes of firms:

- **\( z = 1 \): borrowing firms:** that finance part or all their investment with stocks and/or bonds:
  \[
  M^j(t) + A^j(t) < I^j(t).
  \] (1)

- **\( z = 2 \): hedge firms:** that finance all their investments with internal resources:
  \[
  M^j(t) + A^j(t) \geq I^j(t).
  \] (2)
The investment function for the firm $j$ is given by

$$I_j^z(t) = \alpha h(t) + \beta_z A^j(t) + \epsilon u^j(t)$$  \hspace{1cm} (3)$$

where $I^j$ is the investment, $u$ is the capacity utilization ratio and $\alpha, \beta_z, \epsilon > 0$. The variable $h$ is the valuation ratio (Taylor, 2012), which is the ratio between the values of equity and the value of capital assets in the economy. In the present treatment we set it equal to

$$h(t) = \frac{Pe(t)E(t)}{pK(t)}$$  \hspace{1cm} (4)$$

where $Pe$ is the stock price, $p$ is the final good price and $K$ the aggregate stock of capital assets.

The formulation if the investment function recalls the one in Delli Gatti et al. (1993) where the sensitivity to internal finance analytically devices the Minskyan borrower’s and lender’s risk. In their work the price of equity works as the Tobin $q$. Following Fazzari et al (1988) and Delli Gatti et al (1993), we assume that $\beta_1 > \beta_2$, that is borrowing firms are more sensitive to internal finance as they face the risk of bankruptcy and change their behavior in order to minimize it. Equation (3) involves four factors: a macro-effect ($h$), a meso-effect ($\beta_z$), a micro-effect ($\alpha^j$) and a variable combining micro and macro effects ($u^j$).

All firms adopt the same Leontief-type technology with constant coefficients to produce a homogeneous good that can be used for consumption or investment. As a consequence, the demand for labor at full capacity can be residually quantified once the stock of capital is determined by investment decisions in the previous periods. The supply of labor is infinitely elastic. Accordingly the production function $F$ gives the potential output $\bar{Q}^j$ for firm $j$

$$\bar{Q}^j(t) = F(K^j(t), L^j(t))$$  \hspace{1cm} (5)$$

with $K$ and $L$ representing, respectively, physical capital and labor. Even if excess capital may exist, the output-labor ratio is constant, so that firms are

\footnote{For computational needs, in the multi-agent simulations we consider a sequential economy that evolves in discrete time. For this equation investment depends on the previous period quantities so that $I^j_{z,t} = \alpha h_{t-1} + \beta_z A^j_{t-1} + \epsilon u^j_{t-1}$.}
assumed not to hire excess labor. Since technology exhibits fixed coefficients, it is then possible to define the potential output only as a function of capital so that

$$\bar{Q}^j(t) = 1/\gamma K^j(t)$$  \hspace{1cm} (6)

where the inverse of the capital productivity $\gamma$ is a constant parameter.

The degree of capacity utilization $u^j$ of each firm is defined as the ratio of actual output $Q^j$ sold by the firm to potential output $\bar{Q}^j$, being equal to one at full capacity and smaller than one with excess capacity, so that

$$u^j(t) = \frac{Q^j(t)}{\bar{Q}^j(t)} = \gamma \frac{Q^j(t)}{K^j(t)} \leq 1$$  \hspace{1cm} (7)

Hence, with $\gamma$ as a positive parameter, fluctuations in the degree of capacity utilization of the firms will track changes in the actual output-to-capital ratio.

The quantity actually sold by a firm is subject to a preferential attachment shock. It comes from the assumption that the total demand $pQ(t)$ is known, as it amounts to the sum of consumption and investment, but its distribution among firms needs to be identified. We assume that this distribution is partially stochastic. In particular, the demand is allocated on the base of the relative size of firms (proxied by their capital) to which a stochastic idiosyncratic shock $s$ is added. The expected market share of firm $j$ is

$$\mathbb{E}[Q^j](t) = Q(t) \frac{K^j(t)}{K(t)}$$  \hspace{1cm} (8)

where $Q(t)$ is the total demand, given by the consumption of managers and wage earners, and $K$ represents the aggregate capital for the whole economy. Defining $\tilde{s}$ as a uniformly distributed stochastic variable with $\mathbb{E}[\tilde{s}] = 0$, we set

$$s^j(t) = \tilde{s}^j(t) \left[ 1 - \frac{K^j(t)}{K(t)} \right]$$  \hspace{1cm} (9)

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2 As described in Dutt (1984), the asymmetry which allows excess capital to exist, but the labor-output ratio to be fixed technologically can be justified based on the simplifying assumption that labor will not be hired if it does not contribute to production, whereas the stock of capital is determined by previous investment decisions and may be held in excess.

3 The capital-output ratio is therefore greater than $\gamma$ when there is excess capacity.
in order for $\sum_{j=1}^{N} Q^j = Q$. Accordingly the quantity actually sold by the single firm is

$$Q^j(t) = \mathbb{E}[Q^j](t) \left[1 + s^j(t)\right]$$

(10)

Following Kalecki (1971), firms are assumed to set the price as a mark-up on the cost of labor, while holding excess capacity:

$$p = \left(1 + \mu\right) \frac{w}{\eta}.$$ 

(11)

With $\mu$ taken as a parameter, the mark-up rate is constant and the labor share of output $\Psi$ is given exogenously:

$$\Psi = \frac{w}{p^{\eta}} = \frac{1}{1 + \mu}$$

(12)

The gross profit share of aggregate output $\Pi$ will then be given by

$$\Pi = 1 - \Psi = \frac{\mu}{1 + \mu}$$

(13)

Each firm’s retained profits are computed as the difference between its gross profits, the net interest it pays and the portion $\Theta$ of net profits (defined as gross profits minus interest payments) which is shared with firm’s managers. Since the flow of gross profits is given by a constant share $\Pi$ of each firm’s output by the mark-up rule, retained profits of each firm are given by

$$A^j(t) = (1 - \Theta) \left[\Pi p Q^j(t) [1 + s^j(t)] - r [B^j(t) - M^j(t)]\right]$$

(14)

A firm fails if $A^j/K^j \leq c$, where $c$ is a constant. The case $c = 0$ corresponds to bankruptcy if the firm is unable to pay interest on bonds without issuing new debt (no Ponzi scheme assumption). The probability for a bankrupted firm of being replaced is directly proportional to the performance of the economy in the previous period.

Any excess of retained profits over investment will be held by the firm in the form of money $m$. The law of motion for the stock of money held by firms is thus given by

$$\dot{M}^j(t) = A^j(t) - I^j(t)$$

(15)


5 Firms are assumed to pay interest on bonds $b$ and receive interest on money deposits $m$ at the same interest rate $r$. 
Whenever the flow of investment desired by the firm is higher than the stock of money it holds plus its retained profits in the period, a firm will seek external finance to cover the difference. In particular a borrowing firm will finance its investment for a share $\varpi$ with debt and the rest by issuing new equities. The law of motion for the accumulated stock of firm’s debt $b$ is then

$$\dot{B}_i(t) = \varpi \left[ I_i(t) - A_i(t) - M_i(t) \right]$$  \hspace{1cm} (16)

The amount of equities for borrowing firms evolves according to

$$\dot{E}_i(t) = (1 - \varpi) \left[ I_i(t) - A_i(t) - M_i(t) \right] / Pe(t).$$  \hspace{1cm} (17)

Borrowing firms that become hedge carry on the quantity of shares previously issued.

2.2 The household sector

The household sector is also divided into two sub-categories, namely that of wage earners (subscript $\Psi$) and profit earners (subscript $\Theta$). As described in the previous subsection, workers earn wages $w$ which add up to a constant share $\Psi$ of total output $Q$. Managers (profit earners) receive a share $\Theta$ out of firm’s net profits. All households allocate their total wealth $W$ between money $M$ and shares $E$.

Households’ disposable income $Y$ is composed by wage or profit earnings plus the interest received on the money deposits they hold $M$, so that

$$Y_\Psi(t) = \Psi pQ(t) + rM_\Psi(t)$$  \hspace{1cm} (18)

$$Y_\Theta(t) = \Theta [pQ(t) - rB(t)] + rM_\Theta(t)$$  \hspace{1cm} (19)

The wealth of both classes is accumulated as an effect of savings $S$ and capital gain $G$

$$\dot{W}(t) = S(t) + G(t)$$  \hspace{1cm} (20)

where savings $S$ are defined as the difference between households’ disposable income and consumption levels $S(t) = Y(t) - C(t)$, and $G(t') = [P_e(t') - P_e(t)]E(t)$. Finally, consumption spending by each class will be assumed to be a fixed proportion of both disposable income and the capital gains obtained on equity:

$$C_\Psi(t) = (1 - s_\Psi)Y_\Psi(t)$$  \hspace{1cm} (21)
\[ C_\Theta(t) = (1 - s_\Psi)Y_\Psi(t) + (1 - \sigma_\Theta)G(t) \]  

(22)

where \( s_\Psi \) and \( s_\Theta \) are the propensities to save out of workers’ and managers’
disposable income, and \( \sigma_\Theta \) is the propensity to save out of managers’ capital
gains.

The demand of firms’ shares from household is assumed to be positively
dependent on the previous period’s capital gain and negatively dependent on
the interest rate according to the following functional form

\[
\frac{Pe(t)E(t)}{W(t)} = \frac{1}{1 - \exp[\lambda_r r - \lambda_G G(t - dt)]} 
\]

(23)

with \( \lambda_G, \lambda_r > 0 \). The demand for money is residually determined as

\[ M_h(t) = W(t) - Pe(t)E(t) \]

(24)

Given that only profit earners demand for share, we have that

\[ \dot{M}_\Psi(t) = Y_\Psi(t) - C_\Psi(t) \]

(25)

and, accordingly

\[ M_\Theta(t) = M_h(t) - M_\Psi(t) \]

(26)

2.3 The financial sector

The financial sector is considered as an aggregate. It gives loans to firms,
 hence holding bonds as an asset, and creates money deposits endogenously
 as liabilities. The interest rate paid on deposits and on loans is considered
to be the same for simplicity, so that the financial sector does not make any
 profits (net worth is zero). Thus, the total stock of money \( M \) held by both
 households and firms needs to be equal to the stock of bonds \( B \):

\[ B(t) = \sum_j M^j(t) + M_\Psi(t) + M_\Theta(t) \]

2.4 Goods market equilibrium

Total output \( Q \) is divided between aggregate consumption \( C \) and total in-
vestment \( I \):

\[ pQ(t) = (1 - s_\Psi)Y_\Psi(t) + (1 - s_\Theta)Y_\Theta(t) + (1 - \sigma_\Theta)G_\Theta(t) + I(t) \]
After substituting the labor share $\Psi$ and the profit share $\Pi$ from expressions (12) and (13) in (18) and (19) and solving for $Q$ we obtain

$$pQ(t) = \frac{1 + \mu}{s_\Psi - \mu[1 - \Theta(1 - s_\Theta)]}[I(t) + F(t)]$$

(27)

where

$$F(t) = r[(1 - s_\Psi)M_\Psi(t) + (1 - s_\Theta)(M_\Theta(t) - \Theta B(t))] + (1 - \sigma_\Theta)G_\Theta(t)$$

Table 1 provides a visualization of the balance sheets of the productive, household and financial sectors while table 2 illustrates the social accounting matrix for our economy.

3 Firms dynamics

As illustrated in sections 1 and 2, the analytical solution method adopted by this project operates through a reduction in the heterogeneity by grouping the agents into clusters. For each cluster an average firm can be identified. To this aim, in the present treatment we take the average of the relevant micro-variables within the group. This procedure is defined as the mean-field approximation, which essentially involves reducing the vector of observations of a variable over a population to a single value (Brock and Durlauf, 2001).

The following step is presented in subsection 3.1 and concerns the definition of the probabilistic rules for the switching of firms among the different groups. The dynamics of the number of agents in each cluster is assumed to follow a stochastic process of the Markovian type. This class of processes can be analytically described by a master equation which is a stochastic differential equation. The main steps and the outcome of the solution method for the master equation are presented in subsection 3.2. In particular, the master equation solution can be expressed in compact form by an ordinary differential equation plus a stochastic component, given by a Wiener process. Both the ordinary differential equation and the noise term are formulated as functions of the micro-variables that determine the transition of agents between the different groups. This result is then used in subsection 3.3 to derive the laws of motion of the aggregate variables.

6Physical capital in firms’ balance sheets is evaluated at its market price and not at the evaluation price $h$. We do not therefore consider for the moment the effect of capital gains on firms’ balance sheets.
As for the notation, since the population of firms is reduced to just two (one average hedge firm and one average borrowing firm), the superscript $j$ is no longer needed for the firm-level variables.

### 3.1 Transition probabilities

The micro-probability for a firm of transitioning from one state to another is determined by its capacity of satisfying conditions (1) or (2). These conditions can be quantified by expressing them as functions of the idiosyncratic shock $s$, whose distribution is known by assumption. Let us preliminary define the variable $\Gamma_z$ as

$$\Gamma_z(t) = \frac{I_z(t) - M_z(t) - [\Pi p Q_z(t) - r(B_z(t) - M_z(t))]}{(1 - \Theta)\Pi p Q_z(t)} \frac{K(t)}{K(t) - K_z(t)}$$  \hspace{1cm} (28)

Using equations (1), (2), (10) and (14), it is straightforward to demonstrate that a borrowing firms becomes hedge if

$$s(t) \geq \Gamma_1$$  \hspace{1cm} (29)

and a hedge firm becomes borrowing if

$$s(t) < \Gamma_2$$  \hspace{1cm} (30)

The first probability is indicated by $\eta$ while the second by $\zeta$. Accordingly, we can write

$$\eta^j(t) = Pr[s(t) \geq \Gamma_1]$$  \hspace{1cm} (31)

$$\zeta^j(t) = Pr[s(t) < \Gamma_2]$$  \hspace{1cm} (32)

Assuming $s$ to be uniformly distributed in the interval $[-0.5, 0.5]$, we quantify the two probabilities using the cumulative distribution function of $s$ as

$$\eta(t) = \Gamma_1 + 0.5$$  \hspace{1cm} (33)

$$\zeta(t) = 0.5 - \Gamma_2$$  \hspace{1cm} (34)

These probabilities concern the transition of an individual firm from one state to another, thus they can be defined as micro-probabilities. In order to quantify the number of transitions from one state to the other, we need to
weight these probabilities by the number of firms in each state. Consequently, the configurational transition rates (Weidlich, 2000) read as

\[ \omega_+(t) = N_1(t) \eta(t) \] (35)

\[ \omega_-(t) = N_2(t) \zeta(t) \] (36)

The master equation quantifies the dynamics of the probability of having \( N_1 \) firms in state 1 in a given instant, assuming that the numbers of firms in the two states evolve according to a jump Markov process. It can be formulated as the balance equation between the aggregate transition to and from state 1 and expressed as

\[
\frac{dP(N_1, t)}{dt} = \omega_+(t) P(N_1 - 1)(t) + \omega_-(t) P(N_1 + 1)(t) + [\omega_+(t) + \omega_-(t)] P(N_1)(t)
\] (37)

The probability of having a number \( N_1 \) of borrowing firm is given by the probability of transitioning from a number \( N_1 - 1 \) to \( N_1 \) plus the probability of transitioning from a number \( N_1 + 1 \) to \( N_1 \), less the probability of observing a number \( N_1 \) of borrowing firm times the probability of a transition into or from the borrowing state.

### 3.2 Master equation’s solution: stochastic dynamics of trend and fluctuations

The solution method for the master equation introduced by Di Guilmi (2008), developing Landini and Uberti (2008), yields a system of two equations. The first one is an ordinary differential equation which describes the time evolution of the trend of the stochastic process. The second one is a partial differential equation, known as Fokker-Planck equation, whose general solution identifies the probability distribution of the fluctuations around the drift component\(^7\). This solution technique split the state variable in two components (as proposed by Aoki, 2002) according to

\[ N_1 = N \bar{N}_1 + \sqrt{N} v \] (38)

The factor \( \bar{N}_1 \) is the trend and represents the deterministic component; the variable \( v \) is the spread and quantifies the stochastic noise around the trend.

\(^7\)For a full detail of the solution method we refer the reader to Di Guilmi (2008), Chiarella and Di Guilmi (2011) and Landini and Uberti (2008).
The solution derives an equation for each of the components. In particular, the trend evolves according to the following ODE

\[ \frac{d\bar{N}_1}{d\tau} = \eta \bar{N}_1 - (\eta + \zeta)\bar{N}_1^2 \]  

(39)

where \( \tau = t/N \). The solution for the spread yields the Fokker-Planck equation for the noise, whose stationary distribution is given by the following Gaussian distribution

\[ \theta(u) = C \exp\left(-\frac{u^2}{2\sigma^2}\right) \quad \sigma^2 = \frac{\eta\zeta}{(\eta + \zeta)^2} \]  

(40)

The dynamics of \( n_1 \) can be therefore described by

\[ \frac{n_1(t)}{dt} = \eta u - (\eta + \zeta)u^2 + \sigma dV(t) \]  

(41)

where \( dV \) is a stationary Wiener increment and \( \sigma dV \) is the stochastic fluctuation component in the proportion of speculative firms, coming from the distribution \( (40) \).

### 3.3 Mean-field equations

Having identified the dynamics of the numbers of the two types of firms, we can now use the mean-field values of each micro-variable within the sub-population of speculative or hedge firms in order to study the dynamics of the aggregate variables.

The total amount of investment is given by

\[ \dot{K}(t) = \dot{I}(t) = N_1 I_1 + N_2 I_2 = N\phi(t) + N_1[\beta_1 A_1(t) + \epsilon u_1(t)] + N_2[\beta_2 A_2(t) + \epsilon u_2(t)] \]  

(42)

Accordingly, the evolution of debt and the amount of equities of the mean-field speculative firm can be quantified by re-defining equations (16) and (17), respectively, in the following way

\[ \dot{B}_1(t) = \omega [I_1(t) - A_1(t) - M_1(t)] \]  

(43)

\[ \dot{E}_1(t) = (1 - \omega) [I_1(t) - B_1(t) - M_1(t)] / P e(t). \]  

(44)
Consequently, the dynamics of the aggregate debt in the economy is given by
\[
\dot{B}(t) = N_1 \{ \varpi [I_1(t) - A_1(t) - M_1(t)] \} 
\] (45)
The total number of shares evolves according to
\[
\dot{E}(t) = N_1 \{ (1 - \varpi) [I_1(t) - A_1(t) - M_1(t)] / P_e(t) \} 
\] (46)

4 Analysis

As specified in section 1, the model is initially simulated as agent based with full heterogeneity of agents. In other words, the behavioral rules introduced in subsection 2.1 are applied on \( N = 1,000 \) potentially heterogeneous firms. At each time step the agent based model provides the mean-field values of the micro-variables. In this stage heterogeneity is reduced to two types of firms (one borrowing and one hedge) by taking the average of the relevant variables within each group of firms. Subsequently, the system composed by the equations introduced in section 3.3 is numerically analyzed. In particular we identify a benchmark scenario and study the different dynamics generated by shocking the parameters.

In the benchmark scenario the values of the parameters are \( \gamma = 10; \alpha = 0.1; \beta_1 = 0.01; \beta_2 = 0.05; \varpi = 0.5; s_\Phi = 0.05; s_\Theta = 0.7; \sigma_\Theta = 0.75; r = 0.0015; \Theta = 0.8; \epsilon = 1; \lambda_r = 1; \lambda_G = 0.000001; N = 1,000 \).

Figure 1 shows the evolution of aggregate demand, debt and investment for a single simulation. The three variables display the same trend of growth, with short and very regular cycles for demand and investment. The growth of debt is relatively smooth compared to the other series.

4.1 Effects of the heterogeneity of the investment rules

Figure 2 illustrates the evolution of the number of borrowing firms, on a total of 1,000 firms. It adjusts relatively soon to its steady state level of about 400 and then fluctuates quite widely.

In order to investigate the effects of heterogeneity we shock the parameter \( \beta_2 \), which is the elasticity of investment to internal finance for hedge firm. Figure 3 shows that a larger value of this parameter compared to the benchmark case (\( \beta_2 = 0.01 \)) pushes rapidly the steady state of the debt to capital
It is worth noting that in the case $\beta_1 = \beta_2 = 0.05$ the heterogeneity of behavioral rules is eliminated, being the investment equation identical for both type of firms. The impact on the system of $\beta_2$ is confirmed by figure 4 which reveals considerable larger rates of growth for aggregate demand for a higher sensitivity of hedge firms’ investment to internal finance. Changes in the parameter have no relevant effects on the share of borrowing firms.

It is particularly interesting to assess the effect of heterogeneity on the degree of financialization of the system, defined here as the ratio between market capitalization and aggregate demand. Figure 5 plots the dynamics of the ratio between the total market capitalization, measured by $Pe(t) \ast E(t)$, and the aggregate demand for different values of $\beta_2$. In the benchmark scenario (with $\beta_2 = 0.01$), this ratio displays an increasing trend, since the asset price inflation in the long run is larger than the rate of growth of the economy. A larger $\beta_2$ reduces the degree of financialization of the economy, bringing the ratio to a constant level. This steady state level is lower for larger $\beta_2$. For $\beta_1 = \beta_2 = 0.05$ the rate of growth of the economy converges to the rate of growth of equity price, stabilizing the ratio, despite the larger asset price inflation compared to the benchmark scenario. Heterogeneity, and in particular the financial constraint faced by the borrowing firms, proves to be a factor increasing the dependence of the real sector to the financial sector.

Higher values of the elasticity of investment to the evaluation ratio $\alpha$ and to the capacity utilization $\epsilon$ determine a slightly higher debt to capital ratio. The accumulation of debt is also impacted by the bankruptcy threshold $c$. Figure 6 shows that lowering the threshold modifies the steady state of the aggregate leverage. The figure does not report the simulation with $c = 0$ where the ratio spikes to 300.

### 4.2 Changes in the propensities to save, distribution of income and preference for liquidity

In general, the effect of an increase in the propensity to save for the different categories of income is destabilizing for the system. For a propensity to save of wage earners $\sigma_\Psi \geq 0.1$, all the firms default at the beginning of the simulation because, in the early stages, the biggest fraction of aggregate demand is provided by salaries. As we show below this is reversed as the simulation runs.
A larger propensity to save of profit earners has the effect of lowering the aggregate debt to capital ratio, as shown by figure 7 and increasing level and volatility of aggregate demand, as demonstrated by figure 8. This is due to the fact that an increase in savings raises the demand for equities and their price and, through this channel, the level of investment. The bigger weight of internal finance on the investment decision causes the volatility of profits to affect more significantly the volatility of demand. The study of the correlations between the profits and salary bill with aggregate demand highlights the fact that this economy is profit-led.

The reliance of the system on financial income makes growth possible even with a bigger size of the mark-up, and thus a larger share of profits on overall income. Simulations show that the leverage ratio during the transition toward the steady state is lower for larger $\mu$. This result is due not only to the fact that in our system the only debt is business debt, but also to the increased relevance of capital gains in sustaining growth. In fact, figure 9 reveals that a larger share of income for profit drives up the ratio between size of the equity market and aggregate demand. Also the size of the fluctuations of the ratio increases for the higher variance of both equity price and aggregate demand, proving that the system becomes remarkably more volatile. The explanation involves the fact that the propensity to save out of profit is larger than the one for salaries and this increases the demand for equities and, as a consequence, the size of the financial sector with respect to the real sector.

In our stylized economy the paradox of thrift is avoided by the increasing financialization: the increase in investment due to higher evaluation ratio and the fact that consumption is financed for a progressively increasing weight of financial income and this avoids the shortfall of a lower propensity to consume out of profits and capital gains.

5 Concluding remarks

This paper aims to introduce microfoundations in stock flow consistent modeling to study how the interaction between financial and real sector affects growth and business cycle. To this aim the model is formulated in a bottom-up fashion identifying heterogeneous micro-behavioral rules for firms and then deriving the macro-equations for the productive sector using the master equation.
The research presented here is at a preliminary stage but nevertheless it provides some insights about the transmission of shocks between the financial and the real side of the economy. Numerical simulations of the dynamical system highlight the relevance of the heterogeneity in the investment function. In particular, if financially sounder firms are supposed to be less affected by internal finance in their investment decision, the aggregate leverage in the economy is higher in the adjustment phase and can stabilize at a higher steady state. A higher sensitivity to internal finance, even for sounder firms, increases the level of growth of aggregate demand, reducing the dependence of the real sector to the financial sector.

A larger propensity to save in each of the three income categories (salaries, profits and capital gains) has the effect of noticeably increasing the instability of the system and the size of fluctuations. This effect is particularly relevant for the propensity to save out of capital gains, which also appears to be positively related to the aggregate leverage ratio. This result is strictly linked to the growing weight of the financial sector which affects growth, amplitude of fluctuations, distribution of income and role of the preference for liquidity.

The next development of this work concerns a more refined study of the conditions under which bubbles and busts are generated in the present setting. Also, the model will be extended by introducing a variable mark-up, to study the evolution of the shares of income, and the possibility for households to shift between the two categories of profit-earners and income-earners.

Future research will involve a deeper analysis of the analytical solution of the model, to be achieved by the study of the dynamical system composed by the dynamic equations for the aggregate variables. The analysis will set the stage for micro and macro policy experiments. The policies in question can target either the choice of agents or the distribution of business debt and household income in order to avoid explosive macroeconomic dynamics. These experiments will consider a menu of interconnected policy objectives: stabilization of aggregate output dynamics and business cycle, financial sustainability of growth and equity of income distribution.

References


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<td>$Pe E$</td>
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<td>$M_{\Psi} + M_{\Theta}$</td>
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Table 1: Units and sectoral balance sheets
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<td></td>
<td>Current</td>
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<td>$+C$</td>
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<td>Total</td>
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Table 2: Matrix of flows.
Figure 1: Dynamics of aggregate demand, investment and debt.

Figure 2: Dynamics of the number of borrowing firms (total number of firms: 1,000).
Figure 3: Dynamics of the debt/capital ratio for different values of $\beta_2$.

Figure 4: Dynamics of aggregate demand for different values of $\beta_2$. 
Figure 5: Dynamics of equity value to aggregate demand ratio for different values of $\beta_2$. 
Figure 6: Dynamics of the debt/capital ratio for different values of $c$.

Figure 7: Dynamics of the debt/capital ratio for different values of $s_\Theta$. 
Figure 8: Dynamics of aggregate demand for different values of $s_\Theta$.

Figure 9: Dynamics of equity value to aggregate demand ratio for different values of $\mu$. 